

The Money Pump Is Necessarily Diachronic

Johan E. Gustafsson¹ contends that if Davidson, McKinsey and Suppes' diachronic money-pump argument² is valid, so is the synchronic argument Gustafsson himself offers. He concludes that the latter renders irrelevant diachronic choice considerations in general, and the two best-known diachronic solutions to the money pump problem in particular. I argue here that this reasoning is incorrect, and that Gustafsson's synchronic argument is faulty on independent grounds. Moreover, it raises issues of fidelity to the historical text that must be addressed. I conclude that the money pump, and cyclical choice more generally, are necessarily diachronic; and therefore that the two best-known diachronic solutions to the money pump problem remain relevant.

Davidson *et. al.*'s original formulation of the money pump problem runs as follows. Job candidate Mr. S must choose among three different alternatives:

a = full professor at a salary of \$5,000.

b = associate professor at \$5,500.

c = assistant professor at \$6,000.

Mr. S makes a series of three pairwise comparisons, preferring *a* to *b*, *b* to *c*, and *c* to *a*. The series is *countable* in that each alternative entering into the comparison offers a combination of two kinds of quantifiable preference object - job status + salary - that can be assigned a cardinality within the set of natural numbers on the same numerical scale. It is *continuous* in that the alternatives can be arranged on that scale in a descending or ascending series corresponding to their cardinalities. It is *diachronic* in that it is presumed to unfold in a three-part temporal sequence. And the series is *cyclic* in that it violates transitivity:³ if Mr. S prefers *a* to *b* and *b* to *c*, then by transitivity he

¹ "The Irrelevance of the Diachronic Money-Pump Argument for Acyclicity," *The Journal of Philosophy* CX, 8 (August 2013), 460-464. Quotations are paginated in the text.

² Donald Davidson, J. C. C. McKinsey, and Patrick Suppes, "Outlines of a Formal Theory of Value, I," *Philosophy of Science* 22 (1955), 140-160. Quotations are paginated in the text.

³ I use the term advisedly, because Davidson *et. al.* do. Gustafsson's exposition relies on Sen's weaker concept of acyclicity. But first, Davidson *et. al.*'s formulation of the money pump problem predates Sen's proof (Amartya Sen, *Collective Choice and Social Welfare* (San Francisco: Holden-Day Inc., 1970), Section 1*5. "Choice Functions and Quasi-Transitivity," 14-16). Below I offer some reasons why it would have been better for Gustafsson to adhere as closely as possible to the terms of Davidson *et. al.*'s original

prefers a to c . But by hypothesis, Mr. S also prefers c to a . Similarly, if Mr. S prefers b to c and c to a , then by transitivity, he prefers b to a . But again by hypothesis, Mr. S also prefers a to b . Finally, if Mr. S prefers c to a and a to b , then by transitivity, he prefers c to b . But yet again by hypothesis, Mr. S also prefers b to c . The upshot of Mr. S's sequence of three pairwise comparisons, then, is that he prefers both a to b and b to a ; both b to c and c to b ; and both c to a and a to c . If a rational choice is, as Davidson *et. al.* note, one that selects a most-preferred alternative, or one from a set of mutually indifferent most-preferred alternatives – i.e. satisfies the *non-dominated choice principle*, then Mr. S's cyclical preference is irrational; for his three pairwise comparisons imply that he has no most-preferred alternative at all.

Davidson *et. al.* acknowledge that Mr. S's choice problem is one of inconsistency:

[T]he man who believes the adjacent members of the sequence equivalent will exchange x_1 for x_n in a series of equal swaps, and this is inconsistent with the belief that x_1 is better than x_n (146).

The non-dominated choice principle is a rational one because it imposes a consistency requirement (and some other familiar ones) on the chooser's sequence of pairwise comparisons, namely that it preserve transitivity. A cyclical preference ranking is inconsistent, and an inconsistent preference ranking is irrational. That much seems clear.⁴

exposition. Second, Davidson *et. al.*'s use of transitivity is adequate for their analysis of the money pump. Third, Sen's notion of acyclicity is in any case not as much weaker as it may seem. See Yasuhito Tanaka, "On the topological equivalence of the Arrow impossibility theorem and Amartya Sen's liberal paradox," *Applied Mathematics and Computation* 181 (2006), 1490-1498; I prove the logical equivalence of transitivity and acyclicity in *Rationality and the Structure of the Self, Volume II: A Kantian Conception* (Berlin: <http://adrianpiper.com/rss/docs/PiperRSSVol2KC.pdf> , 2008; 2nd edition 2013), Chapter III. "The Concept of a Genuine Preference," Section 6.2.1. "Occasional Truth Tables for Subsential Constituents." Fourth, acyclicity has by no means superseded the central axiomatic role of transitivity in the contemporary literature. See Peter Fishburn, "Transitivity," *Review of Economic Studies* 46 (1979), 163-173; "Nontransitive Measurable Utility," *Journal of Mathematical Psychology* 26 (1982), 31-67; "Transitive Measurable Utility," *Journal of Economic Theory* 31 (1983), 203-317; and "On Nonstandard Nontransitive Additive Utility," *Journal of Economic Theory* 56 (1992), 426-433. For a philosophical analysis, see John Broome, "Rationality and the Sure-Thing Principle," in *Thoughtful Economic Man*, edited by Gay Meeks, Cambridge University Press, 1991, pp. 74-102.

⁴ But appearances can be deceptive. It is no accident that the term "consistency" does not appear anywhere in Gustafsson's discussion. Despite considerable effort, Ramsey-Savage decision theory has not been successful in establishing a settled formal meaning of this term in the context of rational choice; I review the history of its attempts in *Rationality and the Structure of the Self, Volume I: The Humean Conception* (Berlin: <http://adrianpiper.com/rss/docs/PiperRSSVol1HC.pdf> , 2008; 2nd edition 2013),

But in case it is not, Davidson *et. al.* introduce the notion of the money pump, “in which the point becomes obvious (*ibid.*):” The cynical department head offers Mr. S the associate professorship over the assistant professorship in exchange for a bribe, which the latter delivers. Then he offers Mr. S the full professorship over the associate professorship in exchange for a second bribe, which the latter also duly pays. Lastly, realizing that Mr. S is starting to regret the \$1,000.00 in salary he has sacrificed for the superior status of full professor, the department head demands a third bribe for exchanging Mr. S’s underpaid full professorship for the noticeably more lucrative assistant professorship. Mr. S once again coughs up. By implication, the sequence can be reiterated so long as Mr. S’s finances permit, or until he settles on a most-preferred alternative once and for all.

Thus as Gustafsson rightly notes (462), Davidson *et. al.* intend the envisioned spectacle of the cynical department head, repeatedly pumping Mr. S for bribes so that the latter can regain a preference alternative he had previously rejected, as at best an example that is intended to illustrate the irrationality of cyclical choice behavior over time, by attaching a price to each such irrational choice. They neither claim nor imply that money pumping is identical to cyclical choice over time. Mr. S’s choices would be irrational whether he permitted himself to be pumped or not, and for the same reason. So Gustafsson is correct to argue that it is not the prospect of financial exploitation or loss itself that makes Mr. S’s choice behavior irrational, but rather its inconsistency (461, 463).

On this basis, Gustafsson then reasons that since it is

choosing against one’s preference that is taken to be irrational, the sequential part of the argument is unnecessary. The department head could offer Mr. S a single choice between all three of *a*, *b* and *c*. This ... will force Mr. S to choose an alternative over which another is preferred, which the non-dominated choice principle rules out as irrational (*ibid.*).

“The sequential part of the argument” that Gustafsson maintains is unnecessary is the part that stipulates that Mr. S is presented with a temporally unfolding sequence of three pairwise comparisons. Hence from

Chapters III. “The Utility-Maximizing Model of Rationality: Informal Interpretations,” and IV. “The Utility-Maximizing Model of Rationality: Formal Interpretations.” One solution is now to go to great circumlocutory ends to avoid it altogether. Kotaro Suzumura’s work on consistency (*Consistency, Choice, and Rationality* (Cambridge, Mass.: Harvard University Press, 2010), co-authored with Walter Bossert) proposes a different one. Also see Richard Bradley, “A Note on Incompleteness, Transitivity and Suzumura Consistency,” unpublished paper, 2013. I offer an alternative solution in *Rationality and the Structure of the Self, Volume II: A Kantian Conception*, Chapter III. “The Concept of a Genuine Preference” (*ibid.*).

the correct assumption that the money pumping character of Mr. S's choice is irrelevant to identifying its irrationality, Gustafsson concludes that the diachronic character of his choice is equally irrelevant.

Gustafsson's synchronic argument is based on an analogy: Just as we can derive a synchronic ordinal ranking $\{a, b, c\}$ of all three alternatives from observation of a diachronic sequence of pairwise comparisons that obeys transitivity, similarly we can envision Mr. S himself creating such a synchronic ranking of all three alternatives *ab nuovo*, in a single act of choice. From this Gustafsson infers that Mr. S can similarly create a synchronic ordinal ranking that violates transitivity, *ab nuovo*, in a single act of choice. But this is not so.

Gustafsson stipulates a single choice occasion, in which the department head offers Mr. S all three alternatives simultaneously. However, if Mr. S has only a *single* choice among a, b and c to make, whence comes the description of each one of these three alternatives among which he is to choose as one "over which another is preferred"? There is no prior cyclical ordering inherently built into the three alternatives a, b and c with which Mr. S is presented, by which a preferred other could be determined. In the synchronic scenario, the cynical department head presents the three alternatives neutrally and without bias, and it is up to Mr. S to rank all of them simultaneously. We have just seen that a countable, continuous and transitive ordering of these three alternatives is clearly possible, both diachronically and synchronically.

If this is so, then Mr. S himself must have made the mutually inconsistent choices that produced an *intransitive* ordering of these alternatives, and any further selection behavior is redundant. But those mutually inconsistent choices cannot themselves be part of the synchronic scenario, for they violate its stipulation that Mr. S is offered only "a single choice between all three of a, b and c ." If he makes only a single choice among these three alternatives, then the alternative he chooses is his most-preferred alternative. In this case he orders all three alternatives only once, and none of the possible orderings he can produce violate transitivity: Either he ranks all three alternatives in some descending order; or else he most-prefers one to the other two between which he is mutually indifferent; or else he is indifferent to all three. If, on the other hand, he is presented with a single choice among orderings of the three alternatives $\{a, b, c\}, \{a, c, b\}, \{b, c, a\}, \{b, a, c\}, \{c, a, b\}, \{c, b, a\}$, then it remains the case that any such ordering he chooses satisfies transitivity.⁵ In either case, there is nothing in the synchronic choice scenario that can "force Mr. S to

⁵ Only if Mr. S were offered a choice among combinations of orderings $\{\{a, b, c\} + \{b, c, a\}\}, \{\{a, c, b\} + \{c, b, a\}\}, \{\{c, a, b\} + \{a, b, c\}\}, \dots$ could he violate transitivity with a single choice. But this case would contradict Davidson *et. al.*'s hypothesis, for Mr. S then would not be susceptible to being pumped.

choose an alternative over which another is preferred" – quite the contrary. The non-dominated choice principle remains inviolate.

Gustafsson couches in the passive voice the description of each one of the three alternatives with which Mr. S is presented, i.e. as one "over which another is preferred." But this formulation obscures the background premise that Mr. S's choice itself is the only factor that can determine the preference ordering of *a*, *b* and *c*. Rephrasing the point in the active voice, thus: "This ... will force Mr. S to choose an alternative over which he prefers another," makes this premise salient. But since there is no available criterion of preference independent of or predating the behavioral criterion satisfied by Mr. S's single choice itself, it is not logically possible for him to choose "an alternative over which he prefers another." For in the synchronic scenario, the alternative he most prefers is determined solely by the single choice he makes.

This point assumes the theory of revealed preference, according to which preference is revealed in choice behavior. Since choice behavior occurs in temporal sequence, the preferences it reveals are also revealed in temporal sequence; and it is not easy to imagine a single behavioral event that might reveal two mutually inconsistent preferences.⁶ But even if Gustafsson rejects the theory of revealed preference, Davidson *et. al.* certainly did not. It is of course open to Gustafsson to join those who later contested this theory.⁷ But in that case, he would still need to provide the alternative account of preference that explained how, for any alternative on offer in the synchronic scenario, he could choose that one and only that one alternative, while most-prefering some other alternative that he had not previously chosen. For if he had not previously chosen it, his preference ordering would not be cyclical after all. It would seem, then, that synchronic choice excludes cyclicity. Since the money pump is merely an example of cyclicity, synchronic choice excludes the money pump as well.

If the mutually inconsistent choices that enable Mr. S in the synchronic scenario to "choose an alternative over which he prefers another" cannot themselves be part of that scenario, they must predate it. In that case, Mr. S is choosing on more than one occasion, and the scenario is diachronic rather than synchronic. Under these circumstances, he could certainly most-prefer a different alternative on each such occasion, as a cyclical ranking would require: for example, *a* the first time and *b* the second. Or he could produce

⁶ In *Rationality and the Structure of the Self, Volume I: The Humean Conception*, Chapter III. "The Utility-Maximizing Model of Rationality: Informal Interpretations" (*op. cit.* Note 4), I suggest that this is more than sufficient reason to reject the theory of revealed preference.

⁷ In particular Amartya Sen, "Behavior and the Concept of Preference," *Economica* 40 (1973), 241-259 and his "Rational Fools: A Critique of the Behavioral Foundations of Economic Theory," *Philosophy and Public Affairs* 6, 4 (1977), 317-44.

more than one preference ordering: for example, $\{a, b, c\}$ the first time and $\{c, a, b\}$ the second. In that case, cyclicity is possible but – as we have already seen – not necessary. Mr. S can violate the non-dominated choice principle only by making a diachronic, cyclical series of pairwise comparisons. It would seem, then, that cyclicity presupposes diachronic choice. If cyclical choice is necessarily diachronic, then since the money pump is merely an example of cyclicity, the money pump is necessarily diachronic as well.⁸

Now Gustafsson describes the money pump as the standard “argument” for the consistency of rational preferences (460); and Schick’s⁹ and McClennen’s¹⁰ counterproposals as “objections” to that “argument” (461, 462) – both of which Gustafsson claims are irrelevant (463). Schick’s is claimed to be irrelevant because synchronicity purportedly renders otiose the need for retrospective re-evaluation of cyclical behavior. McClennen’s is claimed to be irrelevant because synchronicity purportedly renders otiose the need for prospective resolve to avoid it. So if Gustafsson’s synchronic argument were correct, both retrospective evaluation and prospective resolve would be irrelevant to the consistency of rational choice. Transitivity then would be back under the yoke of the *separability condition*, which requires the agent to view the alternatives available at each particular moment as separate from earlier choices and from their present and future consequences, i.e. to choose *ab nuovo* on each choice occasion. This would return us full circle, so to speak, to Strotz’s 1955 criticism of the myopia that threatens the theory of expected

⁸ The concept of a diachronic pairwise comparison has been a building block *sine qua non* for the construction of a transitive preference ordering in rational choice theory since its inception. See F.P. Ramsey (1926) “Truth and Probability”, in Ramsey, *The Foundations of Mathematics and other Logical Essays*, Ch. VII, 156-198; edited by R.B. Braithwaite (London: Kegan, Paul, Trench, Trubner & Co.; New York: Harcourt, Brace and Company, 1931), esp. 176-180; John von Neumann and Oskar Morgenstern, *Theory of Games and Economic Behavior* (Princeton, N.J.: Princeton University Press, 1944), Section 3.3. “Probability and Numerical Utilities,” 17-19; Leonard Savage, *The Foundations of Statistics* (New York: Dover Publications, Inc., 1954 and 1971), Section 2.6. “The simple ordering of acts with respect to preference,” 17-21; Kenneth J. Arrow, “Rational Choice Functions and Orderings,” *Economica* XXVI, 102 (May 1959): 121-127; Amartya Sen, *Collective Choice and Social Welfare*, *op.cit.* Note 3. For a contemporary philosophical exposition that takes this concept for granted, see John Broome, *Ethics out of Economics* (Cambridge: Cambridge University Press, 1999), Sections 1.4. “The Formal Techniques,” 2.2. “Axiomatic Utility Theory,” and 2.3. “Expected Utility Theory,” 8-9, 21-26.

⁹ Frederick Schick, “Dutch Bookies and Money Pumps,” *The Journal of Philosophy* LXXXII, 2 (February 1986): 112-119; at 118.

¹⁰ Edward F. McClennen, *Rationality and Dynamic Choice: Foundational Explorations* (New York: Cambridge University Press, 1990), 13.

utility maximization with triviality¹¹ – the very same criticism that spurred development of dynamic choice theory in the first place.

However, Gustafsson's choice of descriptive terms obscures the point and significance of Schick's and McClennen's proposals. Davidson *et. al.* presented cyclical choice behavior in general, and the hapless Mr. S in particular, not as an argument for rational consistency but rather as a *problem* for it, i.e. as an objection to and difficulty for their stipulated axiom of transitivity (143, 145). Both Schick's and McClennen's proposals offer *solutions to the problem* the money pump represents. Both solutions aim to rule out cyclicity, as being incompatible with rational choice. Schick proposes to solve the problem by stipulating that Mr. S review the past, notice the pattern of inconsistency in his previous choice behavior, and discontinue it in the future. McClennen proposes to solve the problem by stipulating that Mr. S at the time of choice resolve to abide by that choice in the future, and in the future to abide by the resolve he made earlier at the time of choice. Since Schick's solution takes for granted that Mr. S's discovery of his past inconsistency can, in fact, motivate an effective resolve to discontinue it in the future, Schick's solution implicitly presupposes the theory of resolute choice that McClennen subsequently developed. Both proposals assign a crucial and inherently diachronic role to remembering the past and planning the future accordingly. An agent that lacks these abilities is not capable of rational choice under any description.¹²

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¹¹ R. H. Strotz, "Myopia and Inconsistency in Dynamic Utility Maximization," *The Review of Economic Studies* 23, 3 (1955 - 1956), 165-180. For a careful statement of the separability condition, see McClennen, *ibid.* 120-122.

¹² I defend a conception of rational agency that builds in these abilities and avoids cyclicity in *Rationality and the Structure of the Self, Volume II: A Kantian Conception*, Chapter III. "The Concept of a Genuine Preference" (*op. cit.* Note 4).